

Weighted complete intersections

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One of the central topics of research in birational geometry are Fano varieties – varieties with ample anticanonical bundle. They play a crucial role in Minimal Model Program and present a rich geometric picture. Fano varieties are central in Mirror Symmetry — many of constructions of mirror duality are either Calabi–Yau manifolds or for Fano varieties.

Classification problem for smooth Fano varieties goes back to XIX century. Due to Riemann, the only Fano curve is a projective line. Smooth Fano surfaces, called del Pezzo surfaces, are classified by Pasquale del Pezzo; there are 10 families of them. Fano threefolds are classified by Iskovskikh in 1979 for Picard rank one case and by Mori–Mukai a bit later; there are 105 families of them. There is no classification in higher dimensions; moreover, the number of families (proven to be finite) is expected to be very large.

The most interesting case of Fano varieties are ones with Picard rank one, that is whose second cohomologies are just \mathbb{Z} . Complete intersections in Grassmannians and toric varieties provide the main examples of smooth Fano varieties. The case of complete intersections in Grassmannians is clear; in the case of complete intersections in toric varieties the toric varieties should be weighted projective spaces. Thus it is challenging to study weighted complete intersections, that are zeros of systems of weighted homogenous polynomials in weighted projective spaces.

We start lectures from introducing basic definitions and facts about weighted projective spaces and complete intersections therein. In particular, we get boundness result for families of smooth Fano weighted complete intersections for any given dimension. Then we proceed with studying their Hodge complexity measuring complexity of Hodge diamond of varieties. We study extremal, from this point of view, cases, and discuss their connection with semiorthogonal decompositions of the weighted complete intersections. Finally, we discuss Mirror Symmetry for the complete intersections. We present a construction of their Landau–Ginzburg models and discuss when the construction works. Finally we study invariants of the Landau–Ginzburg models and their connections with invariants of the initial Fano varieties.