EQUIVARIANT COHOMOLOGY OF WEIGHTED FLAG VARIETIES

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These lectures aim to give a flavour of a fascinating interplay between Geometry, Algebra and Combinatorics. In the course of these lectures we will perform Schubert Calculus on weighted flag varieties. We will give the torus-equivariant cohomology of these spaces along with a combinatorial description. The lectures (apart from the basics) are based on joint work with Shaheen Nazir and Imran Qureshi [ANQ].

In these lectures we will cover relevant basics from Lie algebras and their Representation theory. We will recall flag varieties and define weighted flag varieties (WFV). The notion of a WFV is the weighted projective analogue of the flag variety, and was first introduced by Grojnowski, Corti and Reid in [CR]. The WFVs are projective algebraic varieties with quotient singularities. We will restrict our attention to WFVs of Lie type A. That is, the weighted homogeneous varieties corresponding to Lie groups $GL(n, \mathbb{C})$ for a given positive integer n. We will describe Schubert cells on these varieties and corresponding Schubert cycles that give the equivariant cohomology of the WFV.

At the end we aim to give a combinatorial description of these Schubert cycles following Goresky, Kottwitz & MacPherson [GKM]. They defined "equivariantly formal spaces" as spaces whose G-equivariant cohomology can be computed by restricting attention to fixed points and one-dimensional orbits of the maximal torus inside a Lie group G. Thus the combinatorial description is generally referred to as the GKM-description of equivariant cohomology.

These lectures only require basic background on groups, vectorspaces and manifolds.

REFERENCES

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